Harmonic Tension and Musical Phrases

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Musical compositions and the phrases which make them up are often qualitatively described as increases and releases of tension. This "tension" results from our psychological interpretation of the music. The factors which contribute to the amalgam of characteristics which we call "tension" include the melodic direction of the composition, whether the composition obeys our expectations, and the consonance and dissonance of the chords and different note interactions. The first two characteristics are psychological in nature. They depend on our experience and the way in which we interpret music. The third however, the consonance/dissonance or "harmonic tension" of chords and note combinations can be analyzed quantitatively. This analysis, applied to a Bach chorale, shows that even by itself this characteristic of harmonic tension reflects the increase and decrease of tension typically associated with a composition.

The following equation is the basis of this analysis. It assigns a harmonic tension to 2 harmonics ($\text{frequency}_{\text{harmonic\_1}} > \text{frequency}_{\text{harmonic\_2}}$) of different notes based on their frequencies and their harmonic number (i.e. which harmonic they are: fundamental = 1^{st}, 2^{nd}, 3^{rd} etc . . .):

$$\text{harmonic interaction} = \frac{\left(\text{frequency}_{\text{harmonic\_1}}\right) - \left(\text{frequency}_{\text{harmonic\_2}}\right)}{\left(\frac{\text{frequency}_{\text{harmonic\_1}}}{\text{frequency}_{\text{harmonic\_2}}}\right)^{12} \cdot \sqrt{\text{harmonic\_number}_{\text{harmonic\_1}}} \cdot \sqrt{\text{harmonic\_number}_{\text{harmonic\_2}}}}$$

The numerator is the beating frequency between the pitches; the 12^{th} power term in the denominator has the property that it equals $2^{\text{interval in half-steps}}$; and, the square-root terms weight the harmonic tension based on the numbers of the harmonics.

- Harmonic tension varies directly with the numerator or the beating frequency. The faster the beating the more dissonant the sound in general.

- The ratio of frequencies raised to the 12^{th} power accounts for the fact that as the pitches become further apart and as the beats get faster, they are perceived as less dissonant (i.e. the beats get so fast that we no longer hear them as beats)

- Together, the two above properties create an equation where, as the two pitches approach each other, the harmonic tension approaches zero, and where, as the two pitches get further away, the harmonic tension approaches zero. This reflects the way in which we perceive interactions between harmonics.

- Finally, the square-root terms reflect the tendency for higher harmonics to be less significant than lower ones (softer in terms of volume). This way and interaction between a fundamental and a 1^{st} overtone is regarded as more significant than an interaction between a 3^{rd} overtone and a 5^{th} overtone, just as we hear it.

However, in a four-note chord (of which this Bach chorale is almost exclusively composed) , there are many of these interactions between harmonics. A value of the harmonic tension for a four-note chord of this type is given by the following algorithm:

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harmonic_tension(soprano, alto, tenor, bass) := | running_total ← 0
                                                | for i ∈ 1.. 7
                                                |   for j ∈ 1.. 7
                                                |     running_total ← running_total + harmonic_interaction(tenor, bass, i, j)
                                                | for i ∈ 1.. 7
                                                |   for j ∈ 1.. 7
                                                |     running_total ← running_total + harmonic_interaction(alto, bass, i, j)
                                                | for i ∈ 1.. 7
                                                |   for j ∈ 1.. 7
                                                |     running_total ← running_total + harmonic_interaction(soprano, bass, i, j)
                                                | for i ∈ 1.. 7
                                                |   for j ∈ 1.. 7
                                                |     running_total ← running_total + harmonic_interaction(alto, tenor, i, j)
                                                | for i ∈ 1.. 7
                                                |   for j ∈ 1.. 7
                                                |     running_total ← running_total + harmonic_interaction(soprano, tenor, i, j)
                                                | for i ∈ 1.. 7
                                                |   for j ∈ 1.. 7
                                                |     running_total ← running_total + harmonic_interaction(soprano, alto, i, j)
                                                | running_total
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This algorithm calculates the "harmonic interaction" between all combinations of a note's harmonics and the harmonics of notes above it. This is to say that it compares the fundamental of the bass note to the harmonics of the tenor, the harmonics of the alto and the harmonics of the soprano. Then it compares the 2nd harmonic of the bass to the tenor's harmonics, the alto's harmonics and the soprano's harmonics. This is repeated for all harmonics of the bass note. Then it is repeated for the tenor and all its harmonics, and then for the alto and all its harmonics. The sum of all these values are returned as the harmonic tension. The above example, and calculations in this lab are based on the first seven harmonics of each note.

This produces a function which takes four frequencies (soprano, alto, tenor and bass) and returns a single value representative of how consonant/dissonant the chord is.

When applied to all chords in this Bach chorale the following values are calculated:

The dark lines in the graph represent the ends of phrases in the music. Each phrase is characterized by an overall increase of tension and then a release on or just before the final chord. This is consistent with the acoustic quality of the piece. Each phrase is given direction by an increase in tension and then a release of that tension. The values of harmonic tension of each chord reflect this, on the whole.

Also of note is that the piece in its entirety suggests an overall increase in tension and release just before the end. Thus, in the same way phrases develop through an increase and subsequent release of tension, the composition as a whole develops through and increase and subsequent release of tension.

Shifting to the smaller scale, each phrase is composed of two significant peaks, the second of which tends to be higher or more prolonged than the first. Again, this is justified, because each major phrase is divided into two smaller phrases musically. Each pair of these smaller phrases combine to form each of the four major phrases which in turn combine to form the composition.

Examining these values on any smaller scale yields little useful information. When considering only two chords or any other small grouping of chords, our perception of tension is more strongly influenced by melodic direction and whether our expectations are realized or not. However, in the larger scale picture of the composition, the values of harmonic tension quantitatively reflect the shape which we give the piece qualitatively.

A significant source of error with this sort of analysis is that it seeks to take a quantity, namely harmonic tension, which we judge based on many different elements, and examine only one of these elements, neglecting all others. This is why this analysis is effective on a large scale but is inconsistent with regard to details. When it comes to these details, our sense of musical convention and other criteria govern our interpretation of the composition.

It must be said, however, that to a small extent this algorithm reflects melodic direction. This is because the same type of chord (i.e. a major chord, minor etc . . .) will yield a higher harmonic tension if transposed to higher pitches and a lower value if transposed to lower pitches. This is because in the harmonic interaction equation, if pitches are simply transposed the denominator remains the same and the numerator changes depending on the direction of the transposition. In the same way harmonic tension tends to follow a trend of increasing through the phrase and then decreasing at the end, melodically, phrases will tend to follow this same pattern, reinforcing the trend in the data.

Another short-coming of this analysis is that the algorithm, although, justified musically and physically, is not thoroughly tested nor refined. Changes to this algorithm could improve its overall results while preserving those aspects of it which made it successful in this analysis.

On the whole, this analysis illustrated that there is in fact a calculable physical quantity which reflects our notions of tension and release in music. It successfully reflected an increase and decrease in tension through phrases and the piece as a whole.